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F & H MONOPOLES

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ABSTRACT

Supersymmetric monopoles of the heterotic string theory associated with arbitrary non-negative number of the left moving modes of the string states are presented. They include H monopoles and their T dual partners F monopoles (ALE instantons). Massive $F = H$ monopoles are T self-dual. Solutions include also an infinite tower of generic T duality covariant non-singular in stringy frame F&H monopoles with the bottomless throat geometry. The massless $F = -H$ monopoles are invariant under combined T duality and charge conjugation converting a monopole into anti monopole.

All F&H monopoles can be promoted to the exact supersymmetric solutions of the heterotic string theory since the holonomy group is the compact $SO(9)$. The sigma models for M^8 monopoles, which admit constant complex structures, have enhanced world-sheet supersymmetry: (4,1) in general and (4,4) for the left-right symmetric monopoles. The space-time supersymmetric GS light-cone action in monopole background is directly convertible into the world-sheet supersymmetric NSR action.

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1 Introduction

Supersymmetric gravity has been studied extensively over some number of years. The properties of the electrically charged supersymmetric solutions have been compared with the properties of the states in string theories [1], [2]. The results of such comparisons indicate that many of the electrically charged solutions have the interpretation as the string states. Much less is known about the magnetically charged solutions. From the point of view of the low-energy effective actions of supergravity theories, electric as well as magnetic and dyon configurations come out as solutions of semi-classical non-linear equations. None of these soliton-type solutions is directly and unambiguously related to a linear system of excitations describing the quantum states of the string theory. However, the supersymmetric string theories and electrically charged solutions seem to have a particular knowledge about each other. The predictions about the properties of BPS string states sometimes were obtained in the framework of soliton solutions and sometimes vice-versa. One of the most striking examples of such predictions was the one from the string theory. The massless states with $N_L = 0$ (where N_L is the number of left moving modes) were expected to describe the T-self-dual point of the theory [3]. Indeed, the corresponding “solitons” with the vanishing ADM mass were found to be a T-self-dual solutions of the supergravity theory [4]. In addition, they were identified with $N_L = 0$ states of the heterotic string theory [4], [5] [6].

The massless magnetic monopoles are also very interesting solutions, however not much is known about them since the magnetic solutions have not been identified directly with excitations of any kind of a linear system. The best information comes from the conjectured S-duality which tell us that S-dual partners of massless electric solutions exist. The asymptotic form of magnetic solutions of the heterotic string with $N_L \geq 1$ was found in [1]. The complete multi center magnetic solutions have been found recently [7] in the form of the T-covariant magnetically charged solutions of the heterotic string, defined by 28 harmonic functions. They have one half of unbroken space-time supersymmetry of the heterotic string theory.

The interplay between the space-time supersymmetries and the world-sheet supersymmetries for the BPS states was first studied in connection with the heterotic instantons and solitons by Callan, Harvey and Strominger [8]. The analysis was performed for the five-branes and related to them H monopoles [9], [10]. Now the large variety of more general magnetic solutions is available for which the interplay between the space-time and the world-sheet supersymmetries was not studied yet.

The purpose of this paper is to study the generic class of the monopole solutions of the heterotic string theory. This means that we would like to find the exact ten-dimensional supersymmetric solutions which become monopoles of the four dimensional theory upon dimensional reduction. Some of them are expected to be massive, some massless.

One of the purpose of such uplifting of the four-dimensional monopoles was to study the issue of anomaly related α' corrections to these monopoles and the corresponding world-sheet supersymmetric sigma models. Another reason was to understand better the massless monopoles. There was no information available about the behavior of the massless monopoles under T duality. Moreover, the T self-dual solutions in this class is already known [11]: the uplifted $a = 1$ extreme massive magnetic black holes have such property. It seemed unlikely that both $a = 1$ as well as

the massless black holes can be both T self dual. Thus, we wanted to clarify what happens with massless monopoles under T duality.

So far three types of monopole solutions of the heterotic string theory with half of unbroken supersymmetry were known to be exact. For all of them the non-trivial part is a 4-dimensional Euclidean manifold or the 10-dimensional manifold with 5 flat spatial directions.

i) The first type, known in the literature as H-monopoles was first discovered by Khuri [9]. The embedding of the spin connection into the gauge group required the presence of the non-Abelian field in the solution. At the time of their discovery these solutions were interpreted as non-Abelian monopoles. Soon after this work Gauntlett, Harvey and Liu [10] have established the relation between these monopole solutions and the five-brane solutions. They have also observed that the exact stringy monopoles of [9] are actually not non-Abelian monopoles since the non-Abelian vector fields fall down faster than the monopole field would. Rather they are monopoles of a $U(1)$ group resulting from the compactification of the antisymmetric tensor field B , which explains why these solutions are called H-monopoles. The world-sheet supersymmetry of this solution, including the non-Abelian fields, is known to be (4,4) which provides the proof of the absence of α' corrections [8], [12]. The relation between H-monopoles and extreme $a = \sqrt{3}$ magnetic black holes was realized in [13].

ii) The second type of known monopoles [14] if treated in the same spirit has the right to be called F-monopoles. Those solutions are stringy instantons with a constant dilaton, vanishing 3-form H and self-dual curvature in the four-dimensional Euclidean subspace of the five-dimensional Minkowski geometry. They represent the Asymptotically Locally Euclidean (ALE) gravitational instantonic backgrounds coupled to gauge instantons through the so-called “standard embedding”. These solutions were found to be T-dual partners of the H-monopoles [14]. The non-Abelian fields are required to be present in the solution for the exactness. The relevant non-Abelian fields also fall down as the dipole rather than a monopole field, only the $U(1)$ field $F = dA$ has a magnetic charge. The $U(1)$ field A originates not in the antisymmetric tensor field but in the non-diagonal component of the metric in the uplifted solution. The world-sheet supersymmetry of this solution, including the non-Abelian fields, was found to be (4,4). These solutions from the point of view of the four-dimensional geometry may be also associated with the extreme magnetic $a = \sqrt{3}$ black holes.

iii) The third type of exact magnetic solutions of the heterotic string is the uplifted $a = 1$ magnetic extreme black holes [15], [16], supplemented by the proper non-Abelian field for the exactness [17]. They were called “exact $SU(2) \times U(1)$ stringy black holes”. Besides one Abelian vector field $U(1)$ they had a non-Abelian $SU(2)$ vector field. These solutions were found to be T self dual [11]. In the spirit of giving to monopoles the name according to the name of the gauge fields with magnetic charges, this solution can be called F=H monopole. The world-sheet supersymmetry of this solution, including the non-Abelian fields, was found to be (4,1) [17], [18] which is sufficient to prove the absence of α' corrections [12].

In short, the first and the second type of monopoles related by T duality are

$$(F_{\text{magn}} = 0, H_{\text{magn}}) \quad \Longleftarrow T \Longrightarrow \quad (F_{\text{magn}}, H_{\text{magn}} = 0) \quad (1)$$

The third one is T self dual

$$(F_{\text{magn}} = H_{\text{magn}}) \quad \Longleftarrow T \Longrightarrow \quad (H_{\text{magn}} = F_{\text{magn}}) \quad (2)$$

This picture of heterotic monopoles is obviously incomplete. One may expect to find supersymmetric monopoles with 6 $U(1)$ fields H and with 6 $U(1)$ fields F for the heterotic string compactified on a 6-dimensional torus. Those are solutions which we have found. Since they have both F and H fields with the proper fall off at infinity, and they interpolate between all three types of monopoles presented above, we call them F & H monopoles. Under the S duality our F & H monopoles transform into the electrically charged solutions. The electrical charge of the F-fields originates in the magnetic charge of the H fields and vice versa. In particular, solutions with only electric F fields become H monopoles and the electric solutions with only H fields become F monopoles.

$$\begin{aligned} (F_{\text{el}}, H_{\text{el}} = 0) &\quad \Longleftarrow S \Longrightarrow \quad (F_{\text{magn}} = 0, H_{\text{magn}}) \\ (F_{\text{el}} = 0, H_{\text{el}}) &\quad \Longleftarrow S \Longrightarrow \quad (F_{\text{magn}}, H_{\text{magn}} = 0) \end{aligned} \quad (3)$$

Having these solutions with unbroken supersymmetry in the leading approximation we may address the problem: which of these solutions are exact? We will find a simple answer: all of them, with $SO(9)$ gauge group for embedding of the spin connection. (For all of those with $SO(8)$ gauge group the enhanced world-sheet supersymmetry takes place). For all solutions which we will find, the non-Abelian part always falls off faster than a monopole. The $SO(9)$ vector field far away from the core falls off as $V^{YM} \sim 1/r^2$ and hence the corresponding field strength as $\sim 1/r^3$, whereas the Abelian field strength F_{ij} and H_{ij} fall off as $\sim \epsilon_{ijk} x^k / r^3$. Therefore the name F & H monopoles remains valid for these solutions even after they have been promoted to the exact one. The world-sheet supersymmetry of F & H monopoles will be found to be at least (4,1) which is sufficient to prove the absence of α' corrections.

Many new features of the F & H monopoles with none of F or H vanishing or equal to each other, can be seen already at the level of one F and one H field, i.e. at the level of the solutions which is non-trivial only on the Euclidean four manifold. In particular we will study the massless monopoles upon uplifting and the issues of T duality for this case and the structure of the non-Abelian vector fields.

The paper is organized as follows. In Sec. 2 we present the most general known to us solution of the heterotic string theory in ten dimensions which is supersymmetric and magnetically charged asymptotically flat solutions upon dimensional reduction to four dimension. At this stage we consider only the leading order string equations and do not study the α' corrections; there are no non-Abelian fields present. However we have 6 $U(1)$ fields H and 6 $U(1)$ fields F as promised. In Sec. 3 we study the issue of the exactness of the general solution. We calculate the spin connection for the uplifted monopole solution and find how to promote it to the exact one. We find that the holonomy group of the spin connections of monopole solutions is $SO(9)$. This comes as a nice surprise since the electric partners of some of our monopoles have a holonomy group in the non-compact part of the Lorentz group [17], [19]. Therefore the issue of exactness of these electric solutions is not clear. However, all magnetic solutions are fine and can be made exact by

supplementing them by the non-Abelian fields. In Sec. 4 we study the world-sheet supersymmetric sigma models. For the most general M^9 monopoles we find $(1,1)$ supersymmetry. To get the extended ones we study the M^8 monopoles and find $(4,1)$ or $(4,4)$ supersymmetry. In Sec. 5 the M^4 monopoles are studied in detail. Finally in the Appendix A we have the spin connections and the details about the holonomy group of the monopoles. In Appendix B we focus on subtleties of a multi-monopole solutions with more than two centers.

2 Heterotic monopoles

The leading order heterotic string equations can be derived from the following Lagrangian (we write the 10d fields with a hat):

$$S \sim \int dx^{10} \sqrt{\hat{G}} e^{-2\hat{\phi}} [\hat{R} + 4(\partial\hat{\phi})^2 - \frac{1}{12}\hat{H}^2] \quad (4)$$

This action is the bosonic part of the pure $d = 10, N = 1$ supergravity. We do not add any Abelian vector fields which would be responsible for the 16 vector multiplets in toroidal compactification of the heterotic string on \mathbf{T}^6 . From this action we would get only 6 vector multiplets in $d = 4$. The reason for not working from the beginning with Abelian vector multiplets in addition to the gravitational multiplet is that we will need the non-Abelian vector multiplets in $d = 10$ to keep supersymmetry with account of quantum corrections. Let us start with the solution of this 10-dimensional theory and later we will discuss the corresponding 4-dimensional theory.

A. Solution in $D = 10$

We assume that the fields depend only on three coordinates (x^i) and denote the internal 6 coordinates by x^α . Thus we are looking for the static solutions with isometries in all 6 internal directions. All fields are constructed out of 12 harmonic functions.

The 10d metric is then given by³

$$\begin{aligned} \hat{ds}^2 &= -dt^2 + e^{-4U} dx^i dx^i + (dx^\alpha + A_i^{(1)\alpha} dx^i) G_{\alpha\beta} (dx^\beta + A_i^{(1)\beta} dx^i) \\ e^{-4U} &= 2(|\vec{\chi}^R|^2 - |\vec{\chi}^L|^2) \quad , \quad G_{\alpha\beta} = \delta_{\alpha\beta} - \frac{2\chi_{(\alpha}^L \chi_{\beta)}^R}{|\vec{\chi}^R|^2 + (\vec{\chi}^R \vec{\chi}^L)} \end{aligned} \quad (5)$$

where $\vec{\chi}^R$ and $\vec{\chi}^L$ define the 12-dimensional harmonic $O(6,6)$ -vector

$$\vec{\chi}(x) = \begin{pmatrix} \vec{\chi}^L(x) \\ \vec{\chi}^R(x) \end{pmatrix} \quad , \quad \partial_i \partial_i \vec{\chi}(x) = 0 \quad . \quad (6)$$

For the dilaton we find

$$e^{-2\hat{\phi}} = e^{2U} \frac{1}{\sqrt{\det G}} = \sqrt{2} e^{4U} \frac{|\vec{\chi}^R|^2 + (\vec{\chi}^R \vec{\chi}^L)}{|\vec{\chi}^R|} \quad (7)$$

³We use the notation $\chi_{(\alpha}^L \chi_{\beta)}^R = \frac{1}{2}(\chi_\alpha^L \chi_\beta^R + \chi_\alpha^R \chi_\beta^L)$ and $\chi_{[\alpha}^L \chi_{\beta]}^R = \frac{1}{2}(\chi_\alpha^L \chi_\beta^R - \chi_\alpha^R \chi_\beta^L)$.

The 10d antisymmetric tensor components are given by ⁴.

$$\begin{aligned}\hat{B}_{\alpha\beta} &= \frac{2\chi_{[\alpha}^L\chi_{\beta]}^R}{|\vec{\chi}^R|^2+(\vec{\chi}^R\vec{\chi}^L)} \\ \hat{B}_{\alpha\mu} &= A_{\mu\alpha}^{(2)} + \hat{B}_{\alpha\beta}A_{\mu}^{(1)\beta}\end{aligned}\tag{8}$$

Both Kaluza-Klein gauge fields are pure magnetic and given by

$$\begin{pmatrix} F_{ij}^{(1)} \\ F_{ij}^{(2)} \end{pmatrix} = \sqrt{2}\epsilon_{ijm}\partial_m \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \vec{\chi}^L \\ \vec{\chi}^R \end{pmatrix} = \sqrt{2}\epsilon_{ijm}\partial_m \begin{pmatrix} \vec{\chi}^L + \vec{\chi}^R \\ -\vec{\chi}^L + \vec{\chi}^R \end{pmatrix}\tag{9}$$

The first six vector field strengths

$$F_{ij}^{(1)\alpha} = \partial_i A_j^{(1)\alpha} - \partial_j A_i^{(1)\alpha} \equiv \vec{F}\tag{10}$$

are build out of the non-diagonal component of the metric $g_i^\alpha = A_i^{(1)\alpha}$. When this magnetic field is present in the solution we will call it F part of the monopole solution. The second set of magnetic vector fields is based on the antisymmetric tensor fields

$$F_{ij\alpha}^{(2)} = \partial_i A_{j\alpha}^{(2)} - \partial_j A_{i\alpha}^{(2)} \equiv \vec{H}.\tag{11}$$

In the simplest case when $\hat{B}_{\alpha\beta}$ or $A_i^{(1)\alpha}$ fields are absent

$$F_{ij\alpha}^{(2)} = \partial_i \hat{B}_{\alpha j} - \partial_j \hat{B}_{\alpha i},\tag{12}$$

as it follows from eq. (8). Thus we will call the non-vanishing magnetic charges in the $F_{ij}^{(2)}$ sector the H part of the solution.

All solutions above have one half of $d = 10$, $N = 1$ supersymmetry unbroken. This follows from the fact that we have found these solutions by performing the supersymmetric uplifting of the BPS solutions of $d = 4$, $N = 4$ theory [7]. For the choice of asymptotically flat configurations which we adopt in this paper, the supersymmetric uplifting can be performed by using the procedure and notation of Maharana-Schwarz theory [20] and developed by Sen [1]. If we would be interested in configurations which are not asymptotically flat, one would have to switch to the more general case of dimensional reduction and use the work by Chamseddine [21]. This would give the correct assignment of the fields to various supersymmetric multiplets.

As was already explained, the reason we call solutions (5) F& H monopoles is the fact that they become magnetically charged solution with two sets of vector fields, F and H, upon dimensional reduction. Specifically, the $O(6,6)$ -invariant bosonic action in the form of Maharana-Schwarz [20] and Sen [1] is

$$\begin{aligned}S &= \frac{1}{16\pi} \int d^4x \sqrt{-\det G} e^{-2\phi} \left[R_G + 4G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{8} G^{\mu\nu} \text{Tr}(\partial_\mu \mathcal{M} L \partial_\nu \mathcal{M} L) \right. \\ &\quad \left. - \frac{1}{12} (H_{\mu\nu\rho})^2 - \frac{1}{4} G^{\mu\mu'} G^{\nu\nu'} F_{\mu\nu}^a (L \mathcal{M} L)_{ab} F_{\mu'\nu'}^b \right].\end{aligned}\tag{13}$$

⁴For the multi monopole solution of the generic type with more than two centers there is a subtlety concerning the status of B_{ik} terms. This will be discussed in Appendix B.

This is a bosonic part of $N = 4$ supergravity interacting with six $N = 4$ vector multiplets. There is one vector field in each vector supermultiplet and 6 vector fields in supergravity multiplet. We are looking for the solutions in which all vector fields are magnetic and there are no axions $H_{\mu\nu\rho} = 0$. The magnetic potentials for 6 graviphotons are χ_α^R and the magnetic potentials for the six vector multiplets are χ_α^L . They are all harmonic functions, as shown in eq. (6).

B. Solution in $D = 4$

The four-dimensional supersymmetric solution corresponding to the uplifted supersymmetric solution in eq. (5) is given by [7]

$$ds_{\text{str}}^2 = -dt^2 + e^{-4U} d\vec{x}^2, \quad e^{-4U} = 2\chi^T L\chi = e^{4\phi} = 2(|\vec{\chi}^R|^2 - |\vec{\chi}^L|^2), \quad (14)$$

$$\mathcal{M} = \mathbf{1}_{12} + 4e^{4U} \begin{pmatrix} \chi_\alpha^L \chi_\beta^L & \chi_\alpha^L \chi_\beta^R \\ \chi_\alpha^R \chi_\beta^L & \xi \chi_\alpha^R \chi_\beta^R \end{pmatrix}, \quad \begin{pmatrix} F_{ij}^{(L)} \\ F_{ij}^{(R)} \end{pmatrix} = 2\epsilon_{ijm} \partial_m \begin{pmatrix} \vec{\chi}^L \\ \vec{\chi}^R \end{pmatrix}$$

where $\xi = \frac{|\vec{\chi}^L|^2}{|\vec{\chi}^R|^2}$. The canonical four-dimensional metric is

$$ds_{\text{can}}^2 = -e^{2U} dt^2 + e^{-2U} d\vec{x}^2. \quad (15)$$

The one center solutions can be taken in the simplest form

$$\vec{\chi}^L = \frac{\vec{P}_{\text{vec}}}{2|\vec{x}|}, \quad \vec{\chi}^R = \frac{\vec{n}}{\sqrt{2}} + \frac{\vec{P}_{\text{gr}}}{2|\vec{x}|}, \quad \vec{n}^2 = 1, \quad \vec{n} \cdot \vec{P}_{\text{gr}} \geq 0 \quad (16)$$

For this spherically symmetric solution $|\vec{x}| = \sqrt{(x^1)^2 + (x^2)^2 + (x^3)^2} = r$. It may be useful to rewrite the expressions for \vec{F} and \vec{H} in an explicit form in terms of magnetic charges.

$$\begin{pmatrix} \vec{F}_{ij} \\ \vec{H}_{ij} \end{pmatrix} = \frac{1}{\sqrt{2}} \epsilon_{ijm} \frac{x^m}{|\vec{x}|^3} \begin{pmatrix} \vec{P}_{\text{gr}} + \vec{P}_{\text{vec}} \\ \vec{P}_{\text{gr}} - \vec{P}_{\text{vec}} \end{pmatrix} = \epsilon_{ijm} \frac{x^m}{|\vec{x}|^3} \begin{pmatrix} \vec{P}_F \\ \vec{P}_H \end{pmatrix} \quad (17)$$

where we have defined

$$\vec{P}_F \equiv \frac{1}{\sqrt{2}}(\vec{P}_{\text{gr}} + \vec{P}_{\text{vec}}) \quad (18)$$

$$\vec{P}_H \equiv \frac{1}{\sqrt{2}}(\vec{P}_{\text{gr}} - \vec{P}_{\text{vec}}). \quad (19)$$

Using S duality [1], [7] one can convert the monopole solutions into electrically charged ones. The electric solution is given by the following formula

$$ds_{\text{str}}^2 = -e^{4U} dt^2 + d\vec{x}^2, \quad e^{4U} = e^{4\phi}, \quad E_i^{(a)} = \frac{1}{2} e^{4U} (\mathcal{M}L)_{ab} \partial_i \chi^b, \quad (20)$$

where U and \mathcal{M} are defined in eq. (14) and $E_i^{(a)}$ is the electric field. $E_i^{(a)} = F_{ti}^{(a)}$. One can see from this formula that the reason why the vector multiplet charge changes the sign during S duality is the following. The asymptotic value of the matrix $\mathcal{M}L$ is

$$\mathcal{M}L = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} . \quad (21)$$

Therefore the upper part related to vector multiplets χ^L changes the sign whereas the lower part related to χ^R does not change the sign. It follows that the magnetic charges of the graviphotons become the electric charges of the graviphotons

$$\vec{P}_{\text{gr}} \Longleftarrow S \Longrightarrow \vec{Q}_{\text{gr}} \quad (22)$$

and the magnetic charges of the vector multiplets become electric charges of the vector multiplets with the opposite sign

$$\vec{P}_{\text{vec}} \Longleftarrow S \Longrightarrow -\vec{Q}_{\text{vec}} \quad (23)$$

Thus S duality trades F for H fields and vice versa.

$$\vec{F}_{\text{magn}} \Longleftarrow S \Longrightarrow \vec{H}_{\text{el}} \quad (24)$$

$$\vec{H}_{\text{magn}} \Longleftarrow S \Longrightarrow \vec{F}_{\text{el}} \quad (25)$$

We have presented in [7] the classification of the monopoles via the classification of their S dual electric partners for which the relation to the elementary string excitation is available [1], [2]. Since all solutions which we consider are supersymmetric the right-moving oscillator modes have $N_R = \frac{1}{2}$. For the left-moving part we obtain

$$N_L - 1 = \frac{1}{2}(Q_{\text{gr}}^2 - Q_{\text{vec}}^2) \quad M^2 = \frac{1}{2}Q_{\text{gr}}^2 . \quad (26)$$

The electric solution (20) describes the following states:

- 1) $N_L = 0$ massive and massless white holes
- 2) $N_L = 1$ extremal $a = \sqrt{3}$ black holes.
- 3) $N_L \geq 2$ discrete set of extremal black holes (for $M^2 = N_L - 1$ they reduce to $a = 1$ black holes).

Our magnetic configurations (5) can be also associated with various values of N_L via the relation (we consider one center solution here)

$$N_L - 1 = \frac{1}{2}(Q_{\text{gr}}^2 - Q_{\text{vec}}^2) = \frac{1}{2}(P_{\text{gr}}^2 - P_{\text{vec}}^2) = (\vec{P}_F \cdot \vec{P}_H) , \quad M^2 = \frac{1}{2}P_{\text{gr}}^2 . \quad (27)$$

- 1) $N_L = 0$ massive and massless monopoles

$$\vec{P}_F \cdot \vec{P}_H = -1 , \quad (P_{\text{gr}}^2 - P_{\text{vec}}^2) = -2 , \quad (28)$$

where the massless limit is given by

$$\vec{P}_F + \vec{P}_H = 0 , \quad \vec{P}_{\text{gr}} = 0 , \quad P_{\text{vec}}^2 = 2 . \quad (29)$$

2) $N_L = 1$ monopoles

$$\vec{P}_F \cdot \vec{P}_H = 0 , \quad P_{\text{gr}}^2 = P_{\text{vec}}^2 . \quad (30)$$

Obviously H monopoles satisfy this constraint since for them $\vec{F} = 0$. However, any solution with $\vec{F} \cdot \vec{H} = 0$ also represents an $N_L = 1$ state. In particular, $\vec{H} = 0$ or any solution with non-vanishing but orthogonal \vec{F} and \vec{H} also belongs to $N_L = 1$ state.

3) $N_L \geq 2$ monopoles

$$\vec{P}_F \cdot \vec{P}_H \geq 1 . \quad (31)$$

In the special case $M^2 = N_L - 1$ they reduce to $a = 1$ extreme magnetic black holes with

$$\vec{P}_F - \vec{P}_H = 0 , \quad \vec{P}_{\text{vec}} = 0 . \quad (32)$$

The remarkable feature of all F & H monopoles with $N_L \geq 2$ was observed in [7]. In the stringy frame in the four-dimensional geometry they are completely non-singular solutions. We get for the metric at $r \rightarrow 0$

$$ds_{\text{str}}^2 = -dt^2 + 2(|\chi^R|^2 - |\chi^L|^2)d\vec{x}^2 \rightarrow -dt^2 + d\rho^2 + (\vec{P}_F \cdot \vec{P}_H)d^2\Omega \quad (33)$$

where $d\rho = \sqrt{\vec{P}_F \cdot \vec{P}_H} dr/r$. Hence, in this limit the 4-dimensional solution is given by a bottomless throat ($M_2 \times S_2$), with the radius squared given by the scalar product of both charge vectors. By identification with the string excitation we find that the radius squared has to be quantized ($\vec{P}_F \cdot \vec{P}_H = (N_L - 1) \geq 1$) and we get the limiting metric in the form

$$ds_{\text{str}}^2 \rightarrow -dt^2 + d\rho^2 + (N_L - 1)d^2\Omega \quad (34)$$

Obviously, for any $N_L = 1$ ($\vec{P}_F \cdot \vec{P}_H = 0$) excitation the radius of the throat vanishes (singularity) and for $N_L = 0$ the throat shrinks to zero already at finite $r = r_c > 0$. The expression for the scalar curvature was calculated in [7]. Using our new notation which focus on the presence of the F and H charges, defined in eqs. (19) we have

$$R_{\text{str}} = \frac{2(4\vec{P}_F \cdot \vec{P}_H)^2 - 8M(4\vec{P}_F \cdot \vec{P}_H)r + 4(6M^2 - (4\vec{P}_F \cdot \vec{P}_H))r^2}{\left((4\vec{P}_F \cdot \vec{P}_H) + 4Mr + r^2\right)^3} . \quad (35)$$

Looking on the denominator of this expression one can see that the solution is non-singular for all positive $(\vec{P}_F \cdot \vec{P}_H)$. In terms of dual string states this scalar product has to be a positive integer defined by $(\vec{P}_F \cdot \vec{P}_H) = (N_L - 1)$ for $N_L \geq 2$. The maximum curvature for these solutions is reached at $r = 0$ and is equal to

$$R_{\text{str}}(0) = \frac{1}{2(N_L - 1)} . \quad (36)$$

3 Stringy α' corrections to F & H monopoles

There is a strong belief that supersymmetry established at the classical level will be preserved with the account of the quantum corrections in absence of anomalies. However, when anomalies are present the preservation of supersymmetry and the issue of BPS states in general are not clear. In some particular situations one can study the quantum corrections to the supersymmetry transformations which are due to anomalies. This has been worked out specifically for the anomaly related α' corrections in the heterotic string theory. The relevant supplement to the 10-dimensional action (4) includes the Yang-Mills field and the generalized curvature coupling (with torsion)

$$\alpha' (Tr(R_-)^2 - Tr F^2) \quad (37)$$

where the Trace operation in the $Tr(R_-)^2$ is over the non-compact Lorentz group $SO(1,9)$ and the one in the Yang-Mills $Tr F^2$ is over the compact $SO(32)$ or $E_8 \times E_8$. The BPS configuration which solves the leading order equations supplies the information about the generalized curvature, which gives the first term in eq. (37). When the generalized ten-dimensional curvature $R_{AB}{}^{CD}$ has a nonvanishing value in the non-compact direction of the Lorentz group, i.e.

$$R_{AB}{}^{0D} \neq 0 \quad (38)$$

there is no possibility to use the standard procedure of the spin embedding into the gauge group. However, when

$$R_{AB}{}^{0D} = 0, \quad (39)$$

the spin connection can have a maximum holonomy group $SO(9)$ which is compact. In such situation one can use the standard procedure of embedding of the spin connection into the gauge group. The advantages of this are

1. The anomaly related α' corrections to the equations of motion in the effective theory, to the action and to the *space-time supersymmetry* rules vanish. The detailed description of the procedure can be found in [22].

2. In terms of the world-sheet supersymmetry embedding of the spin connection into the gauge group means the following: one can start with the (1,0) supersymmetric model and enhance this supersymmetry to (1,1) *world-sheet supersymmetry*. This is believed to be necessary for avoiding chiral anomalies in the left-right asymmetric case. The details of this procedure can be found in [8].

The procedure of correcting the supersymmetric solutions via spin embedding into the gauge group was applied before to many BPS solutions, starting with the symmetric version of the five-brane [8]. The same procedure was also applied to H monopoles [9], to F monopoles [14] and to T self dual monopoles [17]. Typically for all these solutions the Yang-Mills field to be added to the solutions was part of the $SO(4)$ gauge theory. In application to the gravitational waves [22] and to the generalized fundamental strings [23] the corresponding gauge group was found to be $SO(8)$.

It was known however, that the uplifted electrically charged $a = 1$ black holes have the non-compact holonomy group of the generalized spin connections [17]. In more general models, chiral

null models of Horowitz and Tseytlin [19], the holonomy group of the generalized spin connections is also not compact [19] since it includes the non-compact Abelian subgroup of the Lorentz group. For these solutions the possibilities of restoring the unbroken supersymmetry of the classical solution in presence of α' corrections are not clear.

For the generic F & H monopoles the non-Abelian group was not known before, since to find the gauge field one has to calculate the spin connection with account of the torsion. We will first study the general solution and find that the holonomy group of the generalized spin connection is the compact group $SO(9)$. This solution has a non-trivial metric in all 9 directions but time, therefore we will sometimes call it \mathbf{M}^9 . We will describe this most general solution and the corresponding (1,1) supersymmetric sigma model. Afterwards we will focus on the slightly less general solution which can be embedded into $SO(8)$ and whose geometry is non-trivial on \mathbf{M}^8 Euclidean manifold. For this solution we will study the extended supersymmetries on the world-sheet. The reason for this is that to enhance the single world-sheet supersymmetry (1,0) to (2,0) one has to consider a manifold of even dimension and to enhance it to (4,0) one need a manifold of dimension $4n$ with the integer n .

Finally we will perform the detailed study of F & H monopoles with account of α' corrections in the simplest case of \mathbf{M}^4 monopoles which have a non-trivial geometry in the four-dimensional Euclidean space. The relevant gauge group will be again the $SO(4)$.

In this section we will have to introduce the set of notations suitable for dealing with vielbeins and spin connections. We will use the base manifold coordinates on \mathbf{M}^{10} and denote them $x^M = \{t; x^i = (x^1, x^2, x^3); x^\alpha = (x^4, \dots, x^9)\}$. The tangent space will be introduced via the zehnbeins

$$\hat{E}^A = \hat{E}^A_M dx^M, \quad \hat{E}^A = \{e^0; e^i; E^a\} \quad i = 1, 2, 3; a = 4, \dots, 9 \quad (40)$$

The uplifted monopoles metric (5) can be rewritten in the form:

$$ds^2 = -(e^0)^2 + (e^i)^2 + (E^a)^2 = \hat{E}^A \eta_{AB} \hat{E}^B \quad (41)$$

where

$$e^0 = dt \quad (42)$$

$$e^i = e^{-2U} dx^i \delta_{\underline{i}}^i \quad (43)$$

$$E^a = E^a_\alpha (dx^\alpha + A^{(1)\alpha}_{\underline{i}} dx^{\underline{i}}) \quad (44)$$

and the tangent space metric is $\eta_{AB} = \{-1, +1, \dots, +1\}$. The explicit expressions for the six-dimensional vielbein E^a_α and its inverse E^α_a is defined in terms of our 12 harmonic functions and can be found in the Appendix A. Thus it is clear from eq. (42) that the most general monopole solution in this class has a non-trivial nine-dimensional Euclidean manifold \mathbf{M}^9 . The spin connection one-form will be defined as

$$W_{AB} \equiv W_{AB,C} \hat{E}^C \quad (45)$$

with the standard definition of $W_{AB,C}$ in terms of zehnbeins and its derivatives. To build the generalized spin connections we need also the tangent space 3-form H_{ABC} .

The objects of interest are the torsionful spin connections

$$\Omega_{\pm AB} = (W_{AB,C} \pm H_{ABC})E^C \equiv W_{AB} \pm H_{AB} \quad (46)$$

Our tangent space group is the Lorentz group $SO(1,9)$ with 45 generators, 9 of them are boosts B^I and 36 are $SO(9)$ rotations M^{IJ} :

$$M^{AB} = \{B^I \equiv M^{[0I]}, M^{IJ}\} , \quad I, J = 1, \dots, 9. \quad (47)$$

The boosts generators $B^I = \{M^{[0i]}, M^{[0a]}\}$ are responsible for the non-compact directions whereas $M^{IJ} = \{M^{[ij]}, M^{[ab]}, M^{[ia]}\}$ are responsible for the $SO(9)$ rotations. In principle the generalized spin connections may take values in any part of the Lorentz group, in compact part as well as in a non-compact one. We have found that for all F & H monopoles both the metric spin connections W_{AB} as well as torsion part of the spin connection $H_{AB} \equiv H_{ABC}E^C$ take values only in the $SO(9)$ rotation part of the Lorentz group.

$$\Omega_{\pm 0I} = 0 \quad \Omega_{\pm IJ} \neq 0 . \quad (48)$$

The holonomy algebra of the generalized spin connections is the algebra generated by the M^{IJ} and the holonomy group is $SO(9)$. We have performed the explicit calculation of the metric part of the spin connections adapting to our case the well known formulas of Scherk and Schwarz [24]. The $SO(9)$ Yang-Mills one-form field is given by the non-vanishing components of Ω_{-IJ} spin connection

$$V_{IJ} = V_{IJ,K}E^K = \Omega_{-IJ}$$

The details of the calculation and the expression for $\Omega_{\pm IJ}$ can be found in the Appendix A. The net result for the Yang-Mills vector field is

$$V_{IJ,0} = 0 \quad V_{IJ,K} = (W_{IJ,K} + H_{IJK})$$

The first two indices are the indices of the $SO(9)$ gauge group and the third one is a space-time index (in the tangent frame). The zero space-time component of the tangent space vector field vanishes for all solutions: this means that the non-Abelian field is also of a magnetic nature. It is therefore tempting to find out if the $SO(9)$ field $V_{IJ,K}$ carries any magnetic charge. For this purpose we note that far from the core of the monopole (we study the one-center solution here) the vielbeins behave as

$$E^A{}_M \sim c + \frac{c_1}{r} + \frac{c_2}{r^2} + \dots$$

Therefore the metric spin connections, which depends on vielbeins and derivatives of the vielbeins, behave as

$$W_{IJ,K} \sim \frac{d_1}{r^2} + \frac{d_2}{r^3} + \dots$$

The same large distance behavior can be observed for the torsion part of the spin connection. Indeed the curved space 3-form consists of the derivative of a 2-form field which behave as

$$B_{MN} \sim f + \frac{f_1}{r} + \frac{f_2}{r^2} + \dots$$

The 3-form therefore behaves as

$$H_{MNL} \sim \frac{e_1}{r^2} + \frac{e_2}{r^3} + \dots$$

Thus all F & H monopoles have the non-Abelian vector field which comes from the embedding of the spin connection into the gauge group $SO(9)$ and at large distances falls off as

$$V_{IJ,K} \sim \frac{a_2}{r^2} + \frac{a_3}{r^3} + \dots$$

The Yang-Mills field strength would fall off as $\frac{1}{r^3}$. The situation is exactly the same as described in [10] for the H monopoles: there is no magnetic charge associated with the non-Abelian part of the solution, we have only F & H magnetic charges associated with the Abelian vector fields, which we have discussed above.

Since we have established that all F & H monopoles can be supplemented by the non-Abelian $SO(9)$ field via spin embedding one can address the issue of the world-sheet supersymmetry of the heterotic string theory in the generic monopole background.

4 World-sheet actions for \mathbf{M}^9 and \mathbf{M}^8 string monopoles

After the four dimensional monopole solutions have been interpreted as solutions with unbroken supersymmetries of the effective action of the heterotic string theory in critical dimension, one can construct a supersymmetric sigma model in an uplifted monopole target space. The details for the special case of the uplifted $a = 1$ black holes can be found in [18]. The most general monopole solutions with $SO(9)$ non-Abelian gauge field defined by 12 magnetic charges suggest the following $(1, 1)$ supersymmetric sigma model [12].

$$I_{(1,1)} = \int d^2z d^2\theta (G_{\underline{IJ}} + B_{\underline{IJ}}) D_+ X^{\underline{I}} D_- X^{\underline{J}} , \quad (49)$$

where the unconstrained $(1, 1)$ superfield is given by

$$X^{\underline{I}}(x^{\underline{I}}, \theta^+, \theta^-) = x^{\underline{I}}(z) + \theta^+ \lambda_+^{\underline{I}}(z) E_I^{\underline{I}}(x) - \theta^- \lambda_-^{\underline{I}}(z) E_I^{\underline{I}}(x) - \theta^+ \theta^- F^{\underline{I}}(z) . \quad (50)$$

This theory is defined in the Euclidean 9 dimensional manifold \mathbf{M}^9 given by the components of the 9×9 sector of the monopole solution (5), (8).

To find out the class of monopole solutions for which the extended supersymmetry can be established we will limit ourself to the case when one of the directions in the internal 6 manifold is flat. Let $\chi_9^R = \chi_9^L = 0$. This solution is the one defined in eq. (5) but characterized by ten harmonic functions instead of twelve. This monopole solution has very special properties. The non-trivial background is in \mathbf{M}^8 only

$$\begin{aligned} G_{MN} dx^M dx^N &= -dt^2 + (dx^9)^2 + e^{-4U} dx^i dx^i \\ &+ \sum_4^8 (dx^\alpha + A_i^{(1)\alpha} dx^i) G_{\alpha\beta} (dx^\beta + A_i^{(1)\beta} dx^i) \end{aligned} \quad (51)$$

The 10d antisymmetric tensor components $B_{MN} = (B_{\alpha\beta}, B_{i\alpha})$, $\alpha = 4, \dots, 8$ are given in eq. (8).

We may rewrite the geometry (51) of the \mathbf{M}^8 monopoles as the function of ten harmonic functions

$$ds^2 = dudv + G_{\underline{IJ}}(\chi_\alpha^{\underline{L}}, \chi_\alpha^{\underline{R}}) dx^{\underline{I}} dx^{\underline{J}} \quad \underline{I}, \underline{J} = 1, \dots, 8 \quad u = -t + x^9, \quad v = t + x^9 \quad (52)$$

$$B_{\underline{IJ}} = B_{\underline{IJ}}(\chi_\alpha^{\underline{L}}, \chi_\alpha^{\underline{R}}) \quad \alpha = 4, \dots, 8$$

and we may also see that only $B_{\underline{IJ}}$ are non-vanishing and are given in eq. (8). The main property of \mathbf{M}^8 monopole background is: one can prove that the Green-Schwarz and Ramond-Neveu-Schwarz formulation of the superstring are equivalent in this background. The light-cone action of one theory can be transformed into the light-cone action of the other theory by converting $SO(8)$ spinors into $SO(8)$ vectors.

To prove this we will study the conditions of such equivalence, as given by Hull in [25].

i) The non-trivial part of the background in the stringy frame has to describe an 8 dimensional Euclidean manifold

ii) The background has to admit a sufficient number of supercovariantly constant Killing spinors. This allows to construct at least 3 almost complex structures in the right and/or in the left-moving sectors of the theory in which Killing spinors exist.

The equivalence theorem proved by Hull [25] and also the study of supersymmetric sigma models by Hull and Witten [26] and Howe and Papadopolous [12] indicate also that for some backgrounds for which in addition

iii) the Nijenhuis tensor vanishes, one may find the enhancement of supersymmetry from 1 up to 4 in each of the right or left moving sectors of the theory where these complex structures have been found.

All three conditions are met for \mathbf{M}^8 monopoles. We will find that the world-sheet action in the \mathbf{M}^8 monopole background has an extended $(4, 4)$ supersymmetry for all solutions with $N_L = 1$ and $(4, 1)$ for the rest. We may again use the $(1, 1)$ action as given in eq. (49), however now $\underline{I}, \underline{J} = 1, \dots, 8$. Upon integration over the fermionic variables and elimination of the auxiliary fields we get

$$S = \int d\tau d\sigma [(G_{\underline{IJ}} + B_{\underline{IJ}}) \partial_z x^{\underline{I}} \partial_{\bar{z}} x^{\underline{J}} + i\lambda_+^I (\nabla_z^{(+)} \lambda_+)^I - i\lambda_-^I (\nabla_{\bar{z}}^{(-)} \lambda_-)^I$$

$$- \frac{1}{4} R_{IJ, KL}^{(+)} \lambda_+^I \lambda_+^J \lambda_-^K \lambda_-^L + \frac{1}{4} R_{IJ, KL}^{(-)} \lambda_-^I \lambda_-^J \lambda_+^K \lambda_+^L] . \quad (53)$$

The right(left)-handed fermions λ_+^I (λ_-^I) have covariant derivatives with respect to torsionful spin connections Ω_+ (Ω_-). The torsionful curvatures $R_\pm = d\Omega_\pm + \Omega_\pm \wedge \Omega_\pm$ have the exchange properties

$$R_{IJ, KL}^{(+)} = R_{KL, IJ}^{(-)} . \quad (54)$$

For our monopoles with the non-Abelian $SO(8)$ fields the torsionful curvatures are given by the Yang-Mills field strength which due to spin embedding is equal to $R_{IJ, KL}^{(-)}$ and by the gravitational torsionful curvature $R_{IJ, KL}^{(+)}$, which are related to each other by eq. (54).

This action has more of the extended supersymmetries for our monopole solutions. Indeed, one can find the set of three almost complex structures by building them in terms of the bilinear combinations of Killing spinors of our background. The corresponding commuting normalized spinors are $\alpha^{\dot{p}}, \beta^{(m)\dot{p}}, m \leq 7$ where \dot{p} is the spinorial index of the $SO(8)$. The complex structure J_{KL} with all required properties was found in [25] to be

$$J_{KL}^{(m)} = \beta_{\dot{p}}^{(m)} \sigma_{KL}^{\dot{p}\dot{q}} \alpha_{\dot{q}} . \quad (55)$$

Although the number of such almost complex structures can be as large as 7, we are interested only to find at least two, the third one being defined by the two. If there are more than three, the target manifold is reducible. The counting of complex structure proceeds as follows. For solutions with $N_L = 1$ for which the square of the right-handed magnetic charge equals the square of the left-handed magnetic charge, the theory can be embedded into the type II superstring theory (or into $N = 8$ supergravity at the level of the effective four-dimensional action). This solution has left-right symmetry and therefore it has one half of unbroken supersymmetry in both left- and right-handed spinors, i.e. for our $\vec{P}_F \cdot \vec{P}_H = 0$ monopoles we have the double set of almost complex structures. Thus for $N_L = 1$ monopoles one can expect the enhancement of world-sheet supersymmetry from the manifest one in eq. (49) which is $(1,1)$ up to $(4,4)$ if all necessary properties of the complex structures will be established.

For all infinite tower of other solutions with $N_L = 0, N_L = 2, 3, \dots, n$ the left-right symmetry is broken since $P_R^2 = P_L^2 + 2(N_L - 1)$ and therefore $P_R^2 \neq P_L^2$. The space-time Killing spinors exist only in the right moving sector, the left moving one does not have unbroken supersymmetries. These solutions have one half of unbroken supersymmetry of the heterotic string (and only one quarter of type II string). Therefore one can construct the complex structures out of space-time Killing spinors according to Hull's prescription (55) only for the right-moving modes. Therefore, for these solutions the expected world-sheet supersymmetry enhancement is going to be $(4,1)$ under condition that the algebra of these extended supersymmetries closes.

Now that we have established that we do have enough of almost complex structures, since we deal with configurations with unbroken space-time supersymmetries, the crucial question remains whether the commutator of two supersymmetries closes for some of our monopoles or for all of them. This does not seem to be a property of an arbitrary background even with unbroken supersymmetries. The right hand side of the commutator of the first supersymmetry with the one induced by the existence of a covariantly constant almost complex structure J depends of the Nijenhuis tensor

$$N_{IJ}^K = J^L_I J^K_{[J,L]} - J^L_J J^K_{[I,L]} . \quad (56)$$

In this expression comma means a derivative. For a generic solution with unbroken supersymmetry the complex structure J^K_J is covariantly constant, but not necessarily constant, which would force terms like $J^K_{[J,L]}$ to vanish. Therefore the Nijenhuis tensor does not vanish in general and therefore one does not find the enhancement of supersymmetry for any supersymmetric background. However, we have found that for our \mathbf{M}^8 monopole solution the Nijenhuis tensor does vanish, which provide the closure of the algebra of the extended supersymmetries on the world-sheet. The reason is that *all magnetic solutions of the heterotic string have Killing spinors in the stringy frame which are constant*. This property of monopoles provides constant complex structures J^K_J

with $J^K_{[J,L]} = 0$. Indeed, in canonical frame

$$\epsilon^{\text{can}}(x) = e^{\frac{U(x)}{2}} \epsilon_0 , \quad (57)$$

where ϵ_0 is a constant spinor. For all magnetic solutions with $U + \phi = 0$ this means that the covariantly constant spinor in stringy frame is a constant spinor!

$$\epsilon^{\text{str}} = \epsilon_0 \quad (58)$$

This property was observed for the five-branes and H monopoles before [8], [9], where it was also used for the enhancement of world-sheet supersymmetry. For $a = 1$ magnetic black holes it was also found [27] that in the stringy frame the Killing spinors exist globally since they are constant. Now we have verified that for the total family of pure magnetic solutions in the stringy frame the Killing spinors are constant. This can be done for example by observing that the Killing spinor for the $O(6, 22)$ covariant electrically charged solutions was found by Peet [28] to be of the form (57). By S duality it follows that for all magnetic solutions Killing spinors are constant in stringy frame.

Thus \mathbf{M}^8 monopoles (51) with $SO(8)$ non-Abelian gauge fields formulated in the stringy frame have the following properties:

i) non-trivial 8 dimensional Euclidean geometry and unbroken space-time supersymmetry which exists globally: the Killing spinors as well as the complex structures are constant

ii) $N_L = 1$ solutions correspond to sigma models with $(4, 4)$ world-sheet supersymmetry; the GS formulation of the type II superstring theory with manifest space-time supersymmetry is equivalent to the NSR form with the world-sheet supersymmetry: the world-sheet $(1, 1)$ spinor λ^I_+, λ^I_- , which is also an $SO(8)$ vector is converted into the space-time $SO(8)$ spinors $S^q, S^{\dot{q}}$ using the normalized commuting Killing spinors $\alpha^{\dot{p}}, \alpha^p$ of the monopole background:

$$\lambda^I_+ = \alpha^{\dot{p}} \gamma^I_{\dot{p}q} S^q , \quad \lambda^I_- = \alpha^p \gamma^I_{p\dot{q}} S^{\dot{q}} . \quad (59)$$

iii) $N_L = 0, N_L = 2, 3, \dots, n$ solutions correspond to the sigma models with $(4, 1)$ world-sheet supersymmetry; the GS formulation of the heterotic string theory with manifest space-time supersymmetry is equivalent to the NSR form with the world-sheet supersymmetry. However, for these backgrounds only the right-handed space-time supersymmetry is available. All left-handed supersymmetries are broken. Therefore only the right-moving world-sheet spinor can be converted into the space-time spinor $SO(8)$ vector

$$\lambda^I_+ = \alpha^{\dot{p}} \gamma^I_{\dot{p}q} S^q . \quad (60)$$

All solutions in the group $N_L = 2, 3, \dots, n$ are described by the non-singular bottomless throat geometry.

The \mathbf{M}^8 monopoles seem to provide the best laboratory for the exploration of the space-time supersymmetry versus world-sheet supersymmetry.

5 Exact F & H monopoles on M^4

In this section we are going to discuss special examples of our solution for the case that the spatial part is 4-dimensional, i.e. if we have only one non-trivial internal direction, x^4 . Most of the relevant features of F & H monopoles are already present in this case. Similar to the M^8 for special configurations we can here expect an enhancement of the world sheet supersymmetry.

We start with general 5-dimensional solution and then we will consider special examples. For that it is reasonable to rotate our harmonic functions into a new basis by performing the $O(1,1)$ transformation

$$\begin{pmatrix} \chi^{(1)} \\ \chi^{(2)} \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} \chi^L \\ \chi^R \end{pmatrix} \quad (61)$$

The metric and the dilaton are then given by ($y = x^5, \dots x^9$)

$$ds^2 = -dt^2 + d\vec{y}^2 + 4\chi^{(1)}\chi^{(2)}dx^{i2} + \frac{\chi^{(2)}}{\chi^{(1)}} \left(dx^4 + A_i^{(1)}dx^i\right)^2 \quad e^{4\hat{\phi}} = 4(\chi^{(2)})^2. \quad (62)$$

The non-diagonal term in the metric is defined as

$$F = F_{ij}^{(1)} \equiv \partial_i A_j^{(1)} - \partial_j A_i^{(1)} = 2\epsilon_{ijm}\partial_m \chi^{(1)} \quad (63)$$

The nontrivial part of the 3-form field defines the second gauge field

$$H = F_{ij}^{(2)} = \partial_i \hat{B}_{4j} - \partial_j \hat{B}_{4i} = 2\epsilon_{ijm}\partial_m \chi^{(2)} \quad (64)$$

We would like to understand the T duality properties of the uplifted F & H monopoles. For this purpose we will perform Buscher [29] transformation over the solution. The result is very simple: one has to change $\chi^{(1)}$ into $\chi^{(2)}$ and back:

$$\chi^{(1)} \quad \Longleftarrow T \Longrightarrow \quad \chi^{(2)} \quad (65)$$

i.e. the compactification radius g_{44} is inverted and the two gauge fields F&H are exchanged.

The Yang-Mills fields which have to be added to the solutions for avoiding α' corrections to supersymmetry parameterize an $SO(4)$. We will have the space-time indices in a lower position and the Yang-Mills one in the upper position. In the tangent space there are no zero components of the vector field, which means also that there are no time components in the curved space.

$$V_0^{ij} = V_0^{i4} = V_t^{ij} = V_t^{ij} = 0$$

The vector components are

$$V_l^{ij} = e^{2U} \delta_{l[i} \partial_{j]} \left(\ln \chi^{(1)} \chi^{(2)} \right) \quad V_l^{i4} = \frac{1}{2} e^{2U} \epsilon_{lim} \partial_m \left(\ln \frac{\chi^{(1)}}{\chi^{(2)}} \right) \quad (66)$$

The fourth components of the vector fields which become scalars in four dimensions are

$$V_4^{ij} \equiv \Phi^{ij} = -\frac{1}{2} e^{2U} \epsilon_{ijm} \partial_m \left(\ln \chi^{(1)} \chi^{(2)} \right) \quad V_4^{i4} \equiv \Phi^{i4} = \frac{1}{2} e^{2U} \partial_i \left(\ln \frac{\chi^{(1)}}{\chi^{(2)}} \right) \quad (67)$$

Under T duality we have again to exchange $\chi^{(1)}$ and $\chi^{(2)}$ which means that only V_l^{i4} and Φ^{i4} change the sign.

Let us establish the connection with the previously known exact heterotic monopoles ⁵.

A. H monopoles ($N_L = 1$)

For this example the F field (63) is absent,

$$F = 0 . \quad (68)$$

We choose for the harmonic functions

$$\chi^{(1)} = 1/2 \quad , \quad \chi^{(2)} = 1/2 + 1/\sqrt{2} \sum_s \frac{P_s}{|\vec{x} - \vec{x}_s|} . \quad (69)$$

We have taken here the general ansatz as a multi center solution. This, however, is only consistent if we restrict the positions or the charges at the centers. We will come back to this point at the end of this section. Our solution on \mathbf{M}^5 becomes then

$$\begin{aligned} \hat{ds}^2 &= -dt^2 + \mathcal{V}^{-2}(dx^i dx^i + dx^4 dx^4) , \quad F = 0 \\ e^{2\hat{\phi}} &= 2\chi^{(2)} = \mathcal{V}^{-2} \quad , \quad \partial_i \hat{B}_{4j} - \partial_j \hat{B}_{4i} = \epsilon_{ijm} \partial_m e^{2\hat{\phi}} \end{aligned} \quad (70)$$

and only \mathbf{M}^4 is non-trivial. The Yang-Mills fields are

$$V_l^{ij} = -2\delta_{l[i}\partial_{j]}\mathcal{V} \quad V_l^{i4} = \epsilon_{lim}\partial_m\mathcal{V} \quad (71)$$

$$\Phi^{ij} = \epsilon_{ijm}\partial_m\mathcal{V} \quad \Phi^{i4} = \partial_i\mathcal{V} \quad (72)$$

This solution with $N_L = 1$ has self-dual Yang-Mills fields as different from the general case (66), (67).

$$V_l^{i4} = \frac{1}{2}\epsilon_{ikm}V_l^{km} \quad , \quad \Phi^{i4} = \frac{1}{2}\epsilon_{ikm}\Phi^{km} \quad (73)$$

This self-duality is the source of the enhancement of the left-handed supersymmetry, since the integrability condition for the Killing spinors is available. This allows to promote this $N_L = 1$ solution to a supersymmetric solution of the type II string with one half of supersymmetry unbroken. It results also in (4,4) world-sheet supersymmetry for the corresponding sigma model.

B. F monopoles ($N_L = 1$)

This is another example with the same left-handed oscillation number. Now, the H field (64) is absent

$$H = 0 . \quad (74)$$

We take the harmonic functions

$$\chi^{(2)} = 1/2 \quad , \quad \chi^{(1)} = \epsilon/2 + 1/\sqrt{2} \sum_s \frac{P_s}{|\vec{x} - \vec{x}_s|} \quad (75)$$

⁵The M^4 magnetic solution without the non-Abelian part was presented recently in [30].

Now the 3-form and the dilaton is absent and the non-trivial metric on \mathbf{M}^4 is the self-dual multi center metric

$$\hat{ds}^2 = -dt^2 + \mathcal{V}^{-2} dx^i dx^i + \mathcal{V}^2 (dx^4 + \omega_i dx^i)^2, \quad H = 0, \quad e^{2\hat{\phi}} = 1 \quad (76)$$

where

$$\mathcal{V}^{-2} \equiv \frac{\chi^{(1)}}{\chi^{(2)}} = \epsilon + \sum_s \frac{\sqrt{2} P_s}{|\vec{x} - \vec{x}_s|} \quad (77)$$

$$\vec{\nabla}(\mathcal{V}^{-2}) = \vec{\nabla} \times \vec{\omega} \quad (78)$$

This is the multi center Gibbons-Hawking metric [32]. Special cases are: $\epsilon = 0, s = 1$ (one center) which is the flat Minkowski space, $\epsilon = 0, s = 2$ (two center) is the Eguchi-Hanson instanton and for $\epsilon = 1$ this metric corresponds to the multi-Taub-NUT spaces. Again we have certain restriction for the charges or positions of the center (see below).

Under Buscher duality transformations [29] F monopoles are transformed into the H monopoles and back. This is obvious from the fact that under this duality transformation both gauge fields are exchanged.

The F monopoles Yang-Mills fields are

$$V_l^{ij} = -2\delta_{l[i}\partial_{j]}\mathcal{V} \quad V_l^{i4} = -\epsilon_{lim}\partial_m\mathcal{V} \quad (79)$$

$$\Phi^{ij} = \epsilon_{ijm}\partial_m\mathcal{V} \quad \Phi^{i4} = -\partial_i\mathcal{V} \quad (80)$$

This solution with $N_L = 1$ has anti-self-dual Yang-Mills fields as different from the general case (66), (67).

$$V_l^{i4} = -\frac{1}{2}\epsilon_{ikm}V_l^{km}, \quad \Phi^{i4} = -\frac{1}{2}\epsilon_{ikm}\Phi^{km} \quad (81)$$

Here again we are dealing with enhanced supersymmetries. The solution has one half of unbroken supersymmetries in the type II string and on the world-sheet we have a (4,4) supersymmetric sigma model.

C. T self dual monopoles ($N_L \geq 2$)

We know that under T-duality both gauge fields get exchanged. Therefore to have a self dual solution we assume that both gauge fields (63) and (64) are equal ($\chi^{(1)} = \chi^{(2)}$)

$$F = H. \quad (82)$$

It corresponds to uplifted $a = 1$ extreme massive magnetic black holes [11]. In notation of this paper they have $\chi^L = 0$ and

$$\chi^{(1)} = \chi^{(2)} = 1/2 + 1/\sqrt{2} \sum_s \frac{P_s}{|\vec{x} - \vec{x}_s|} \equiv \chi \quad (83)$$

The non-trivial metric on \mathbf{M}^4 and the dilaton are

$$ds^2 = -dt^2 + e^{4\hat{\phi}} dx^{i2} + (dx^4 + A_i^{(1)} dx^i)^2 \quad e^{4\hat{\phi}} = 4(\chi)^2. \quad (84)$$

The Yang-Mills part of the solution is

$$V_l^{ij} = -2\delta_{l[i}\partial_{j]}e^{-2\hat{\phi}} \quad V_l^{i4} = 0 \quad (85)$$

The fourth components of the vector fields which become scalars in four dimensions are

$$\Phi^{ij} = \epsilon_{ijm}\partial_m e^{-2\hat{\phi}} \quad \Phi^{i4} = 0 \quad (86)$$

This is an $SU(2)$ non-Abelian field in agreement with [17]. The Yang-Mills field here is not self-dual. Therefore this solution can be embedded into type II string only as a solution with one quarter of supersymmetry unbroken, since all left-handed supersymmetries are broken. This leads to (4,1) supersymmetry on the world sheet, i.e. the enhancement of supersymmetries takes place only in the right-handed sector of the theory.

D. Massless monopoles ($N_L = 0$)

These solutions are given by $\chi^R = \text{const}$, which means that both gauge fields (63) and (64) differ only by a sign

$$F = -H . \quad (87)$$

Thus, we define our harmonic functions as

$$\chi^{(1)} = 1/2 + 1/\sqrt{2} \sum_s \frac{(P_{\text{vec}})_s}{|\vec{x} - \vec{x}_s|} , \quad \chi^{(2)} = 1/2 - 1/\sqrt{2} \sum_s \frac{(P_{\text{vec}})_s}{|\vec{x} - \vec{x}_s|} \quad (88)$$

The metric and the dilaton are now

$$ds^2 = -dt^2 + 4\chi^{(1)}\chi^{(2)}dx^i{}^2 + \frac{\chi^{(2)}}{\chi^{(1)}} \left(dx^4 + A_i^{(1)}dx^i\right)^2 \quad e^{4\hat{\phi}} = 4(\chi^{(2)})^2 . \quad (89)$$

This solution describes in four dimensions a massless monopole that was uplifted into the 5-dimensional stringy geometry.

Under T duality transformation the massless uplifted monopoles are not invariant since $\chi^{(1)} \neq \chi^{(2)}$. If we would perform Buscher [29] transformation over the solution and combine it with the charge conjugation ⁶

$$\chi^{(1)} \quad \Longleftarrow \quad T_d \quad \Longrightarrow \quad \chi^{(2)} \quad (90)$$

$$(P_{\text{vec}})_s \quad \Longleftarrow \quad C \quad \Longrightarrow \quad -(P_{\text{vec}})_s \quad (91)$$

we would find that the solution is invariant. Indeed, for the massless monopoles

$$\chi^{(1)} \quad \Longleftarrow \quad T_d \times C \quad \Longrightarrow \quad \chi^{(1)} \quad (92)$$

$$\chi^{(2)} \quad \Longleftarrow \quad T_d \times C \quad \Longrightarrow \quad \chi^{(2)} \quad (93)$$

This property of the massless monopoles was not predicted before and the fact that it involves T duality and changing the monopole into the anti-monopole $T_d \times C$ does not seem to follow from any known principles. It is observed here as a property of the explicit solution.

⁶In what follows we will use a notation T_d for T duality since in the context of charge conjugation C one may expect the symbol T to be associated with time reflection.

The YM fields for the massless monopoles are given in eqs. (85), (86) with the harmonic functions defined in eq. (88). There is no special simplifications: the non-Abelian fields belong to the $SO(4)$ gauge group. This solution breaks one half of the supersymmetry of the heterotic string, and corresponds to (4,1) supersymmetric sigma model.

E. The moduli matrix \mathcal{M}

Thus we have listed here four special type of F & H monopoles. Two of them, H monopoles and F monopoles have the four-dimensional projection in which the metric and the dilaton are the same for both solutions and coincide with those of the extreme magnetic $a = \sqrt{3}$ black holes

$$ds_{\text{str}}^2 = -dt^2 + e^{-4U} d\vec{x}^2, e^{-4U} = \chi = e^{4\phi} = 1 + \frac{2M}{r}, \quad M = P/\sqrt{2} \quad (94)$$

The difference comes in moduli \mathcal{M} and in the vector fields since the right-handed harmonic function χ^R is the same for both solutions but the left-handed one differs by the sign of the charge. For our case here the moduli metric \mathcal{M} is given by

$$\mathcal{M} = \begin{pmatrix} \frac{\chi^{(1)}}{\chi^{(2)}} & 0 \\ 0 & \frac{\chi^{(2)}}{\chi^{(1)}} \end{pmatrix} \quad (95)$$

For the H monopole we have therefore for the non-vanishing component of moduli and vector fields

$$\mathcal{M}_H = \begin{pmatrix} \frac{1}{1+\frac{2M}{r}} & 0 \\ 0 & 1 + \frac{2M}{r} \end{pmatrix}, \quad \begin{pmatrix} F_{ij}^{(L)} \\ F_{ij}^{(R)} \end{pmatrix} = \epsilon_{ijm} \partial_m \frac{P}{r} \begin{pmatrix} -1 \\ +1 \end{pmatrix} \quad (96)$$

For the F monopoles the non-vanishing components of moduli and vector fields are

$$\mathcal{M}_F = \mathcal{M}_H^{-1} = \begin{pmatrix} 1 + \frac{2M}{r} & 0 \\ 0 & \frac{1}{1+\frac{2M}{r}} \end{pmatrix}, \quad \begin{pmatrix} F_{ij}^{(L)} \\ F_{ij}^{(R)} \end{pmatrix} = \epsilon_{ijm} \partial_m \frac{P}{r} \begin{pmatrix} +1 \\ +1 \end{pmatrix} \quad (97)$$

The T self dual solution in four dimensional form becomes equal to the extreme $a = 1$ dilaton black hole:

$$ds_{\text{str}}^2 = -dt^2 + e^{-4U} d\vec{x}^2, \quad e^{-4U} = \chi = e^{4\phi} = \left(1 + \frac{2M}{r}\right)^2, \quad M = P/\sqrt{2} \quad (98)$$

The moduli fields are constant and vector fields are only the right-handed ones.

$$\mathcal{M} = \mathbf{1}_{12}, \quad \begin{pmatrix} F_{ij}^{(L)} \\ F_{ij}^{(R)} \end{pmatrix} = \epsilon_{ijm} \partial_m \frac{P}{r} \begin{pmatrix} 0 \\ +1 \end{pmatrix} \quad (99)$$

Finally, the massless monopoles in the four-dimensional world are the ones whose metric and the dilaton are:

$$ds_{\text{str}}^2 = -dt^2 + e^{-4U} d\vec{x}^2, \quad e^{-4U} = e^{4\phi} = 1 - \frac{1}{2} \left(\frac{P_{\text{vec}}}{r} \right)^2, \quad M = 0 \quad (100)$$

There are some non-vanishing components of the moduli fields and the vector fields are only the left-handed ones.

$$\mathcal{M} = \begin{pmatrix} \frac{r+2M}{r-2M} & 0 \\ 0 & \frac{r-2M}{r+2M} \end{pmatrix}, \quad \begin{pmatrix} F_{ij}^{(L)} \\ F_{ij}^{(R)} \end{pmatrix} = \epsilon_{ijm} \partial_m \frac{P_{\text{vec}}}{r} \begin{pmatrix} +1 \\ 0 \end{pmatrix} \quad (101)$$

F. Remark about the multi center case

All our magnetic (Abelian) gauge fields are given by Dirac monopole solutions. As a consequence in order to formulate the solution consistently one has to remove the so-called Dirac-singularities. This is in principle not difficult, one has only to take care that it can be done for all centers simultaneously. We are describing this procedure in detail in the Appendix B. As result we find that for all F & H monopoles i) the position of the centers can be arbitrary, but all charges may differ only by the sign and for the other cases ii) all centers have to line-up. For the second case the charges are arbitrary. By fixing the gauge one could also relax the line-up restriction, but not in a gauge invariant way. The two-center solutions can always be placed on a line, therefore there are no restrictions. The subtlety is relevant starting from three centers.

As we have already pointed out above, we have listed here four particular special cases of F & H monopoles. The singularities of these solutions have been studied before. The $N_L = 0$ massless monopoles and $N_L = 1$ F & H monopoles are singular in four-dimensional geometry. The majority of solutions, i.e. the generic $N_L = 2, 3, 4, \dots$ solutions have an arbitrary non-vanishing values of F and H and are non-singular bottomless holes from the point of view of the four dimensional stringy geometry.

6 Discussion

The large family of monopole solutions described in this paper provides an interesting area of study of non-perturbative effects in supersymmetric gravity. The fact that these configurations have a non-trivial Euclidean geometry on an eight dimensional manifold \mathbf{M}^8 has made these new solutions a most interesting objects realizing the relations between unbroken space-time supersymmetry and world-sheet supersymmetry. The role of $SO(8)$ gauge group with its particularly remarkable relations between vector and spinor representations acquires a new and beautiful aspect when applied to supersymmetric stringy monopoles. The unique property of such monopoles to admit complex structures in the corresponding sigma models is related immediately to the fact that the space-time Killing spinors for magnetic configurations are constant in the stringy frame. Therefore we have observed the enhancement of world-sheet supersymmetries for all stringy monopoles which live on the Euclidean \mathbf{M}^8 manifold. These monopoles, although discovered as a soliton type solutions of the classical field equations of supergravity seem to have the most stable properties concerning the quantum corrections. The supersymmetry in the space-time as well as the supersymmetry at the world-sheet are free of anomalies. The supersymmetric non-renormalization theorems do not have

any obvious obstructions and therefore the \mathbf{M}^8 monopoles give us an example of rather reliable non-perturbative BPS states of the superstring theory.

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7 Appendix A: Spin connections

Now we define the vielbeins. The zehnbein can be written as

$$\hat{E}^A{}_M = \begin{pmatrix} e^0{}_t & 0 & 0 \\ 0 & e^i{}_{\underline{i}} & 0 \\ E^a{}_{\beta} A^{(1)\beta}{}_{\underline{i}} & E^a{}_{\alpha} & 0 \end{pmatrix} \quad (102)$$

The curved three dimensional indices are underlined. The same can be rewritten as

$$\hat{E}^A = \hat{E}^A{}_M dX^M = \{e^0 = dt, e^i = e^i{}_{\underline{i}} dx^{\underline{i}}, E^a = E^a{}_{\beta} A^{(1)\beta}{}_{\underline{i}} dx^{\underline{i}} + E^a{}_{\alpha} dx^{\alpha}\} \quad (103)$$

The corresponding vierbeine are given by

$$e^0{}_t = 1, \quad e^i{}_{\underline{j}} = e^{-2U} \delta^i{}_j \quad (104)$$

The six-dimensional vielbein is defined in terms of our 12 harmonic functions as

$$E^a{}_{\alpha} = \delta^{a\beta} \left[\delta_{\beta\alpha} - \frac{1}{|\vec{\chi}^R|^2 + (\vec{\chi}^R \vec{\chi}^L)} \left((1 - \sqrt{1 - \frac{|\vec{\chi}^L|^2}{|\vec{\chi}^R|^2}}) \chi_{\beta}^R \chi_{\alpha}^R + \chi_{\beta}^L \chi_{\alpha}^R \right) \right] \quad (105)$$

The inverse quantities are:

$$G^{\alpha\beta} = \delta^{\alpha\beta} + 2e^{4U} [\chi_{\alpha}^L \chi_{\beta}^L + \frac{|\vec{\chi}^L|^2}{|\vec{\chi}^R|^2} \chi_{\alpha}^R \chi_{\beta}^R + 2\chi_{(\alpha}^L \chi_{\beta)}^R]$$

$$E^{\alpha}{}_a = \left[\delta_{\alpha\beta} + \frac{\sqrt{2}e^{2U}}{|\vec{\chi}^R|} \left((1 - \sqrt{1 - \frac{|\vec{\chi}^L|^2}{|\vec{\chi}^R|^2}}) \chi_{\alpha}^R \chi_{\beta}^R - \chi_{\alpha}^R \chi_{\beta}^L \right) \right] \delta^{\beta}{}_a$$

The results for the metric spin connection one-form consist of three type of terms representing the $SO(3)$ rotation $M^{[ij]}$, the $SO(6)$ rotation $M^{[ab]}$ and the non-diagonal terms $M^{[ai]}$.

$$W_{ij} = 2(\partial_{[i} e^{2U}) e_{j]} - \frac{1}{2} F_{ij,a} E^a \quad (106)$$

$$W_{ab} = E_{[a}{}^{\alpha} \delta_i{}^{\underline{i}} (\partial_{\underline{i}} E_{\alpha b]) e^i \quad (107)$$

$$W_{ai} = F_{ik}{}^{\alpha} E_{\alpha a} e^k - \frac{1}{2} E_a{}^{\alpha} e^{2U} \delta_i{}^{\underline{i}} (\partial_{\underline{i}} G_{\alpha\beta}) E^{\beta}{}_c E^c \quad (108)$$

All components of spin connection related to the boosts are vanishing:

$$W_{0A} = 0 \quad (109)$$

To calculate the tangent space three form H_{ABC} we will use the expression for the curved space three form and the zehnbeins

$$H_{ABC} = E_A{}^M E_B{}^N E_C{}^L H_{MNL} \quad (110)$$

where $H_{MNL} = 3\partial_{[M} B_{NL]}$ and the components of the two-form B_{MN} are defined in eq. (8)

For all monopoles $H_{tNL} = 0$. It follows that

$$H_{0BC} = 0 \quad H_{0B} \equiv H_{0BC}E^C = 0 \quad (111)$$

We will get 3 types of terms for $H_{ABC}E^C$. Since none of the indices in $H_{ABC}E^C$ takes the value 0 we could rewrite it as $H_{IJK}E^K$. We will get 3 types of terms for it.

$$H_{ij} = H_{ijk}e^k + H_{ija}E^a \quad (112)$$

$$H_{ab} = H_{abk}e^k + H_{abc}E^c \quad (113)$$

$$H_{ai} = H_{aik}e^k + H_{aic}E^c \quad (114)$$

For the one center solution only the term H_{ijk} vanishes, for the multi center solution the situation is more complicated. Thus in general all components of H_{IJ} are non-vanishing.

There are two combinations of spin connection and three form which we need to know

$$\Omega_{\pm IJ} = W_{IJ} \pm H_{IJ} \quad (115)$$

They are both taking values in $SO(9)$ group, in general. For some particular solution they may take values in much smaller groups which are subgroups of $SO(9)$ but they do not need more than $SO(9)$ and since there are no boost components in neither metric spin connections nor in the three forms, this concerns both generalized spin connections $\Omega_{\pm IJ}$.

8 Appendix B: Multi center solutions

Here we are going to discuss possible restrictions for parameters of the multi center solution. We consider for simplicity the \mathbf{M}^4 monopoles presented in Sec. 5. The solution is defined in terms of two harmonic functions. To describe a multi center solution we take these harmonic functions in the form

$$\chi^{(1)} = a^{(1)} + \sum_{s=1}^{k+1} \frac{p_s^F}{|\vec{x} - \vec{x}_s|} \quad , \quad \chi^{(2)} = a^{(2)} + \sum_{s=1}^{k+1} \frac{p_s^H}{|\vec{x} - \vec{x}_s|} \quad (116)$$

where \vec{x}_s are the positions and p_s are the charges of the centers. For the gauge fields in the multi-center case we can also make an ansatz as a sum over different one-center solutions

$$\vec{A}^{(1/2)} = \sum_{s=1}^{k+1} \vec{A}_s^{(1/2)} \quad (117)$$

We are describing here Dirac monopole solutions which are globally not defined. To remove the Dirac singularities one has to introduce for every center two different coordinate patches. In each of them one defines the gauge field without a singularity and finally one has to glue together all patches. For one center the different gauge fields are then given by [31]

$$\vec{A}_s^{(1/2)} = \frac{p_s^{(F/H)}}{2r_s(z - z_s \pm r_s)} ((y - y_s) dx - (x - x_s) dy) = \frac{1}{2} p_s^{(F/H)} (\mp 1 + \cos \theta_s) d\phi_s \quad (118)$$

where $r_s^2 = (x - x_s)^2 + (y - y_s)^2 + (z - z_s)^2$ and θ_s and ϕ_s are the angular variables of the center s . For every gauge field we have a “+” and a “-” part which are singular for $\theta_s = 0$ or π and we have to take the non-singular one in the different patches. In the overlapping region both parts are connected by a gauge transformation which is equivalent to a shift in the x^4 coordinates (see (62) and also [32])

$$x_{(s)}^4 \rightarrow x_{(s)}^4 \mp p_s^F \phi_s . \quad (119)$$

(the “ \mp ” ambiguity comes from the fact that one can approach the overlapping region from two different patches). Since we have to identify the field configuration in the overlapping region we find that x_s^4 has to be periodic with the period of $2\pi p_s^F$ which means that the x^4 direction is a circle with the radius p_s^F . This has to be done for every center. This procedure can be done only if the compactification radius of x^4 is the same for all centers. Otherwise one could not put together all the the different coordinate patches for all centers. But since the period or radius of the x^4 differs if the magnetic charges differs from center to center we get the result that all magnetic charges are the same up to a sign or zero, i.e.

$$p_s^F = p^F \eta_s^F , \quad \eta_s^F = 0, \pm 1 \quad (120)$$

There is a second possibility to remove all Dirac singularities consistently. Namely if all centers are on a line, e.g. if x_s and y_s are equal all centers line-up parallel to the z -direction. Then we can introduce for all centers the same coordinate system with the same angular variable ϕ , $\vec{A}^{(1)} d\vec{x} \equiv A_\phi^{(1)} d\phi$. Consequently, the periodicity condition for x^4 is now given by

$$x^4 \simeq x^4 + P^F \phi \quad (121)$$

where $P^F = \sum_{s=1}^{k+1} p_s^F$, the total magnetic charge. Again the x^4 direction is compactified on a circle with radius proportional to the total magnetic charge.

These results were related to the fact that $\vec{A}^{(1)}$ was the KK gauge field in the metric and thus restricts the p_s^F . A simple T duality, however, exchange both gauge fields $\vec{A}^{(1)}$ and $\vec{A}^{(2)}$ and thus yields the same restrictions for p_s^H as well.

To summarize, in order not to have Dirac singularities in the \mathbf{M}^4 metric we have two possibilities i) all centers have up to the sign the same magnetic charges (or zero) p^F and p^H , or ii) all centers have to line-up. For the harmonic functions for the multi center solutions this means

$$\begin{aligned} i) \quad & \chi^{(1)} = a^{(1)} + p^F \sum_{s=1}^{k+1} \frac{\eta_s^F}{|\vec{x} - \vec{x}_s|} , \quad \chi^{(2)} = a^{(2)} + p^H \sum_{s=1}^{k+1} \frac{\eta_s^H}{|\vec{x} - \vec{x}_s|} \\ ii) \quad & \chi^{(1)} = a^{(1)} + \sum_{s=1}^{k+1} \frac{p_s^F}{|\vec{x} - \vec{z}_s|} , \quad \chi^{(2)} = a^{(2)} + \sum_{s=1}^{k+1} \frac{p_s^H}{|\vec{x} - \vec{z}_s|} \end{aligned} \quad (122)$$

where $\eta_s = 0, \pm 1$. In both cases the x^4 coordinate has to be compact. Obviously, the two-center case falls into the second possibility. The first non-trivial case are three centers.

Now we turn to the question what do these restrictions mean for the torsion or antisymmetric tensor. We start with the calculations of the Chern-Simons term, which is part of the torsion in $D = 4$. Using (116) and (118) we find

$$\begin{aligned} & (A_i^{(1)} F_{jl}^{(2)} + A_i^{(2)} F_{jl}^{(1)} + \text{cycl.perm.}) \sim \\ & \sim \epsilon_{ijl} \left(A_m^{(1)} \partial_m \chi^{(2)} + A_m^{(2)} \partial_m \chi^{(1)} \right) = \\ & = \epsilon_{ijl} \sum_{st} \frac{p_s^F p_t^H + p_s^H p_t^F}{r_s(z - z_s \pm r_s) r_t^3} [(x - x_s)(y - y_t) - (y - y_s)(x - x_t)] . \end{aligned} \quad (123)$$

Since our $D = 4$ torsion has to vanish there are two possibilities. Either the Chern-Simons term and the antisymmetric tensor vanish each or they cancel against each other ⁷. The Chern-Simons term vanishes under the two conditions:

$$\begin{aligned} i) \quad & p_s^F p_t^H = -p_t^F p_s^H \\ ii) \quad & \text{all centers line-up, i.e. } x_s = x_t, y_s = y_t \forall s, t. \end{aligned} \quad (124)$$

The first condition means that either p_s^F or p_s^H is identical zero for all centers, i.e. only F or H monopoles. The line-up condition *ii*) coincides with the condition *ii*) in (122). If these conditions are fulfilled we find that the $D = 4$ antisymmetric tensor has to be zero, too. Also the $D = 5$ antisymmetric tensor (see footnote) is zero, since in the line-up case the gauge fields have only one unique ϕ -component and in the other case one gauge field vanishes. Thus,

$$\hat{B}_{ij} = B_{ij} = 0 \quad , \quad i, j = 1, 2, 3 \quad (125)$$

Finally, we have to investigate the possibility that the antisymmetric tensor part cancels the Chern-Simons part. This is in our case possible since for our gauge fields $F^{(1)} \wedge F^{(2)} = 0$.

$$\begin{aligned} 3\partial_{[m} B_{np]} &= 6(A_{[m}^{(1)} F_{np]}^{(2)} + A_{[m}^{(2)} F_{np]}^{(1)}) \\ &= \epsilon_{mnp} (A_l^{(1)} \partial_l \chi^{(2)} + A_l^{(2)} \partial_l \chi^{(1)}) . \end{aligned} \quad (126)$$

This equation has a solution if the gauge fields are in the Coulomb gauge ($\partial_l A_l^{(1/2)} = 0$) and we find

$$B_{np} = \epsilon_{npl} (A_l^{(1)} \chi^{(2)} + A_l^{(2)} \chi^{(1)}) \quad (127)$$

and inserting this expression into the $D = 5$ antisymmetric tensor (see footnote) yields

$$\hat{B}_{np} = \epsilon_{npl} (A_l^{(1)} \chi^{(2)} + A_l^{(2)} \chi^{(1)}) - 2(A_n^{(1)} A_p^{(2)} - A_p^{(1)} A_n^{(2)}) . \quad (128)$$

This procedure is possible independently on the positions of the centers, but it is somehow unsatisfactory since it is gauge dependent (only for Coulomb gauge). The situation becomes even worse when we remember that the gauge invariance of KK gauge fields is related to general covariance of the 5-dimensional theory (translations in the fourth coordinate). From this point of view it seems to be that the multicenter solution is consistent in 4 as well as embedded in 5 dimensions only i) if one gauge field vanishes (i.e. F or H monopoles) or ii) if all centers line-up. In the last case the charges could be arbitrary whereas in the case of F or H monopoles it is necessary that all charges may differ only by a sign (to remove all Dirac singularities).

⁷Note, the 4-dimensional torsion is given by [20] $H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} - 2(A_\mu^{(1)} F_{\nu\rho}^{(2)} + A_\mu^{(2)} F_{\nu\rho}^{(1)}) + \text{cycl.perm.}$, with $B_{\mu\nu} = \hat{B}_{\mu\nu} + 2(A_\mu^{(1)} A_\nu^{(2)} - A_\nu^{(1)} A_\mu^{(2)})$

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